# KMA315 Analysis 3A: Problems 1

*The problems should be submitted by 4:00pm on Friday the $11^{th}$ of March.*

1. Find the infimum (greatest lower bound) and supremum (least upper bound) of the following subsets of $\mathbb{R}$ (justify your claims):

(i) $\{\frac{2}{n+1} : n \in \mathbb{N}\}$; *(4 marks)*

(ii) $\{\frac{(-1)^n}{n^3} : n \in \mathbb{Z}_+\}$; *(4 marks)*

(iii) $\{n^{(-1)^n} : n \in \mathbb{N}\}$. *(4 marks)*

2. Let $A$ be a subset of real numbers such that $A$ contains only a finite number of elements. Is it possible for the greatest lower bound of $A$ to <u>not</u> be an element of $A$? Justify your claim. *(2 marks)*

3. Determine and explain whether the following sequences are - (a) bounded above/below, (b) monotone increasing/decreasing, (c) convergent and (d) if they converge then also what their limit is:

(i) $((-1)^n)_{n=0}^{\infty}$; *(5 marks)*

(ii) $(\frac{2}{n+1})_{n=0}^{\infty}$. *(5 marks)*

4. Let $(a_n)_{n=0}^{\infty}$ and $(b_n)_{n=0}^{\infty}$ be sequences of real numbers. Prove that:

(i) if $(a_n)_{n=0}^{\infty}$ and $(b_n)_{n=0}^{\infty}$ are bounded above then $(a_n b_n)_{n=0}^{\infty}$ is also bounded above; *(3 marks)*

(ii) if $(a_n)_{n=0}^{\infty}$ and $(b_n)_{n=0}^{\infty}$ are monotone decreasing then $(a_n + b_n)_{n=0}^{\infty}$ is also monotone decreasing; *(3 marks)*

(iii) if $(a_n)_{n=0}^{\infty}$ and $(b_n)_{n=0}^{\infty}$ converge then $(a_n + b_n)_{n=0}^{\infty}$ also converges [also prove that in such a case we have $\lim_{n\to\infty}(a_n + b_n) = (\lim_{n\to\infty} a_n) + (\lim_{n\to\infty} b_n)$]. *(3 marks)*

5. Prove that if a sequence of real numbers is monotone decreasing and bounded below then it converges to its infimum (aka greatest lower bound). *(4 marks)*

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